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Homotopy perturbation method for the nonlinear dispersive $K(m, n, 1)$ equations with fractional time derivatives

Hüseyin Koçak, Turgut Öziş, Ahmet Yıldırım
Department of Mathematics, Science Faculty, Ege University, İzmir, Turkey

Abstract

Purpose – This paper aims to apply He's homotopy perturbation method (HPM) to obtain solitary solutions for the nonlinear dispersive equations with fractional time derivatives.

Design/methodology/approach – The authors choose as an example the nonlinear dispersive and equations with fractional time derivatives to illustrate the validity and the advantages of the proposed method.

Findings – The paper extends the application of the HPM to obtain analytic and approximate solutions to the nonlinear dispersive equations with fractional time derivatives.

Originality/value – This paper extends the HPM to the equation with fractional time derivative.

Keywords Difference equations, Wave physics, Deformation

Paper type Research paper

1. Introduction

The study of solitary solutions of nonlinear equations in mathematical physics plays an important role in soliton theory. Because, nonlinear wave phenomena arise in a wide range of physical and engineering applications such as fluid mechanics, hydrodynamics, solid state physics plasma physics, optical fibers etc. These phenomena are frequently related to wave and dispersive equations. Therefore, explicit solutions to such equations are of primary importance and there is a strong attention to explicit soliton solutions and these solutions may provide much physical information to help researchers to understand the characteristics of the mechanism of the physical models. Fractional differential equations have been caught much attention recently due to exact description of nonlinear phenomena. No analytical method was available before 1998 for such equations even for linear fractional differential equations. In 1998, the variational iteration method was first proposed to solve fractional differential equations with greatest success (He, 1998). Many authors found variational iteration method (VIM) is an effective way to solving fractional equations both linear and nonlinear (Odibat and Momani, 2006; Das, 2008). Momani and Odibat (2007) and Ganji *et al.* (2008) applied the homotopy perturbation method (HPM) to fractional differential equations and revealed that the HPM is an alternative analytical method for fractional differential equations. Momani *et al.* (2008) and Odibat and Momani (2008) compared solution procedure between VIM and HPM.

In this paper we consider the following nonlinear dispersive $K(m, n, 1)$ equation with fractional time derivatives:

$$D_t^\alpha u + (u^m)_x - (u^n)_{xxx} + u_{5x} = 0. \tag{1}$$

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The classic nonlinear dispersive $K(m, n)$ equation first introduced by Rosenau and Hymann (1993) and for certain values of m and n, $K(m, n)$ equation has solitary waves which are compactly supported. Recently, large number of methods were suggested to study the nonlinear dispersive $K(m, n)$ equations, such as Adomian method (Wazwaz, 2002, 2003, 2004; Zhu *et al.*, 2007), Exp-function method (He and Wu, 2006a), variational iteration method (He and Wu, 2006b; Wazwaz, 2007; Tian and Yin, 2007), variational method (He, 2006a, b) and HPM (He, 2005a, b; Odibat, 2007). There are also many effective and convenient methods for solving these type equations (Inç, 2006; Zhu and Lu, 2006; Zhu and Gao, 2006; Tian and Yin, 2005). Equation (1) with fractional order of time derivation happens in discontinuous time in large time scale in weather forecast or the very small time scale in high energy physics. Time is discontinuous according to the E-infinity theory (but Hierarchical), and the fractional model is the best candidate to describe such problems. Time-fractional equations always behave fascinatingly as illustrated (He, 2008c).

Recently, Odibat (2007) has successfully utilized the HPM to construct solitary solutions for nonlinear dispersive $K(m, n)$ equation with fractional time derivatives. In this paper, we would like to extend the HPM to the $K(m, n, 1)$ equation with fractional time derivative given in Equation (1) above.

The objective of this paper is to extend the application of the HPM to obtain analytic and approximate solutions to the nonlinear dispersive $K(m, n, 1)$ equations with fractional time derivatives. The HPM was first proposed by the Chinese mathematician Ji-Huan He (He, 1999, 2000a, b, 2006c). The essential idea of this method is to introduce a homotopy parameter, say p, which takes values from 0 to 1. When $p = 0$, the system of equations usually reduces to a sufficiently simplied form, which normally admits a rather simple solution. As p gradually increases to 1, the system goes through a sequence of deformations, the solution for each of which is close to that at the previous stage of deformation. Eventually at $p = 1$, the system takes the original form of the equation and the final stage of deformation gives the desired solution. One of the most remarkable features of the HPM is that usually just few perturbation terms are sufficient for obtaining a reasonably accurate solution. Considerable research works have been conducted recently in applying this method to a class of linear and nonlinear equations (Öziş and Yıldırım, 2007a, b, c, d; Yıldırım and Öziş, 2007; Yıldırım, 2008a, b, c, d; Dehghan and Shakeri, 2007, 2008; Shakeri and Dehghan, 2007, 2008; Saadatmandi *et al.*, 2009). The interested reader can see the references (He, 2008a, b, 2006d, e) for last development of HPM.

2. Fractional calculus

We give some basic definitions and properties of the fractional calculus theory which are used further in this paper.

Definition 2.1

A real function $f(x),\ x > 0$, is said to be in the space $C_\mu,\ \mu \in R$ if there exists a real number $p(> \mu)$, such that $f(x) = x^p f_1(x)$, where $f_1(x) \in C[0, \infty)$, and it is said to be in the space C_μ^m if and only if $f^{(m)} \in C_\mu, m \in N$.

Definition 2.2

The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$, of a function $f \in C_\mu,\ \mu \geq -1$, is defined as

$$J^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt, \quad \alpha > 0, \quad x > 0, \quad J^0 f(x) = f(x).$$

Properties of the operator J^{α} can be found in references (Miller and Ross, 1993; Samko *et al.*, 1993; Oldham and Spanier, 1974), we mention only the following. For $f \in C_{\mu},\ \mu \geq -1,\ \ \alpha, \beta \geq 0$ and $\gamma > -1$:

(1) $J^{\alpha}J^{\beta}f(x) = J^{\alpha+\beta}f(x)$,

(2) $J^{\alpha}J^{\beta}f(x) = J^{\beta}J^{\alpha}f(x)$,

(3) $J^{\alpha}x^{\gamma} = \Gamma(\gamma+1)/\Gamma(\alpha+\gamma+1)x^{\alpha+\gamma}$.

The Riemann-Liouville derivative has certain disadvantages when trying to model real world phenomena with fractional differential equations. Therefore, we shall introduce a modified fractional differential operator D^{α} proposed by Caputo in his work on the theory of viscoelasticity (Luchko and Gorneflo, 1998).

Definition 2.3
The fractional derivative $f(x)$ in the Caputo sense is defined as:

$$D^{\alpha}f(x) = J^{m-\alpha}D^{m}f(x) = \frac{1}{\Gamma(m-\alpha)}\int_{0}^{x}(x-t)^{m-\alpha-1}f^{(m)}(t)dt, \tag{2}$$

for $m-1 < \alpha \leq m, \quad m \in N, \quad x > 0, \quad f \in C_{-1}^{m}$.

Also, we need here two of its basic properties.

Lemma 2.3.1 If $m-1 < \alpha \leq m, \quad m \in N$ and $f \in C_{\mu}^{m},\ \mu \geq -1$, then,

$$D^{\alpha}J^{\alpha}f(x) = f(x),$$

and,

$$J^{\alpha}D^{\alpha}f(x) = f(x) - \sum_{k=0}^{m-1}f^{(k)}(0^{+})\frac{x^{k}}{k!}, \quad x > 0.$$

The Caputo fractional derivatives are considered here because it allows traditional initial and boundary conditions to be included in the formulation of the problem. In this paper, we consider the nonlinear dispersive $K(m,n,1)$ equations with fractional time derivatives, and the fractional derivatives are taken in Caputo sense as follows.

Definition 2.4
For m to be the smallest integer that exceeds α, the Caputo time-fractional derivative operator of order $\alpha > 0$ is defined as

$$D_{t}^{\alpha}u(x,t) = \frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}} = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)}\displaystyle\int_{0}^{t}(t-\tau)^{m-\alpha-1}\frac{\partial^{m}u(x,\tau)}{\partial t^{m}}d\tau, & \text{for} \quad m-1 < \alpha < m \\ \dfrac{\partial^{m}u(x,t)}{\partial t^{m}}, & \text{for} \quad \alpha = m \in N \end{cases} \tag{3}$$

For more information on the mathematical properties of fractional derivatives and integrals one can consult the mentioned references.

3. The fractional *K* (*m*, *n*, 1) equations

In this section, solitary solutions for two special cases of the nonlinear dispersive $K(m,n,1)$ equations with fractional time derivatives are obtained by HPM.

Example 3.1

The fractional K(2, 2, 1) equation

We first consider the fractional $K(2, 2, 1)$ equation with initial condition:

$$D_t^\alpha u + (u^2)_x - (u^2)_{xxx} + u_{5x} = 0, \quad t > 0 \tag{4}$$

$$u(x,0) = \frac{16c-1}{12}\cosh^2\left(\frac{x}{4}\right), \tag{5}$$

where $0 < \alpha \leq 1$ and c is an arbitrary constant. We construct the following homotopy:

$$D_t^\alpha u - p\{-(u^2)_x + (u^2)_{xxx} - u_{5x}\} = 0, \quad p \in [0,1] \tag{6}$$

$$u_0(x,0) = \frac{16c-1}{12}\cosh^2\left(\frac{x}{4}\right), \tag{7}$$

Assume the solution of Equation (6) to be in the form:

$$u = u_0 + pu_1 + p^2u_2 + p^3u_3 + \ldots\ldots \tag{8}$$

Substituting (8) into (6) and equating the coefficients of like powers of p, we get following set of differential equations

$$\begin{aligned}
p^0: &\quad D_t^\alpha u_0 = 0, \quad u_0(x,0) = \frac{16c-1}{12}\cosh^2\left(\frac{x}{4}\right) \\
p^1: &\quad D_t^\alpha u_1 = -(u_0^2)_x + (u_0^2)_{xxx} - (u_0)_{5x}, \quad u_1(x,0) = 0 \\
p^2: &\quad D_t^\alpha u_2 = -(2u_0u_1)_x + (2u_0u_1)_{xxx} - (u_1)_{5x}, \quad u_2(x,0) = 0 \\
p^3: &\quad D_t^\alpha u_3 = -(2u_2u_0 + u_1^2)_x + (2u_2u_0 + u_1^2)_{xxx} - (u_2)_{5x}, \quad u_3(x,0) = 0 \\
&\ldots
\end{aligned} \tag{9}$$

Solving the above equations, we obtain

$$\begin{aligned}
u_0 &= \frac{16c-1}{24}\left(\cosh\left(\frac{x}{2}\right) + 1\right), \\
u_1 &= -\frac{(16c-1)c}{24.2}\sinh\left(\frac{x}{2}\right)\frac{t^\alpha}{\Gamma(\alpha+1)}, \\
u_2 &= \frac{(16c-1)c^2}{24.2^2}\cosh\left(\frac{x}{2}\right)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}, \\
u_3 &= -\frac{(16c-1)c^3}{24.2^3}\sinh\left(\frac{x}{2}\right)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \\
&\ldots
\end{aligned} \tag{10}$$

and so on, in the same manner the rest of components can be obtained using Maple. Consequently, we have the first five terms (Figure 1) for solution of Equations (4) and (5)

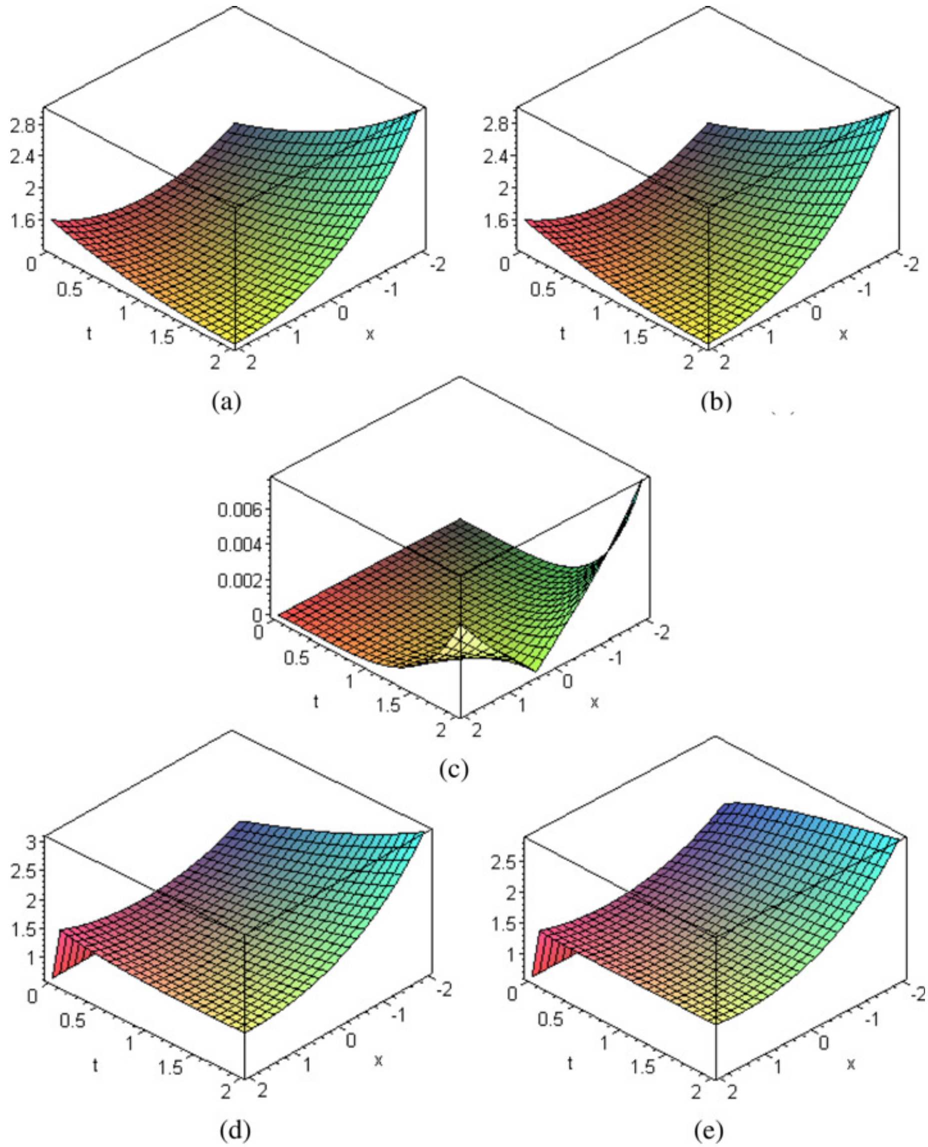

Figure 1.

Notes: $c = 1$; (a) exact solution (12) for $\alpha = 1$, (b) approximate solution (11) for $\alpha = 1$, (c) absolute error for $\alpha = 1$, (d) exact solution (12) for $\alpha = 0.75$ and (e) exact solution (12) for $\alpha = 0.25$

in a series form:

$$u(x,t) = \frac{16c-1}{24}\left[\cosh\left(\frac{x}{2}\right)\left(1+\frac{c^2}{2^2}\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+\frac{c^4}{2^4}\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}+\cdots\right)+1\right] - \frac{16c-1}{24}\left[\sinh\left(\frac{x}{2}\right)\left(\frac{c}{2}\frac{t^{\alpha}}{\Gamma(\alpha+1)}+\frac{c^3}{2^3}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}+\cdots\right)\right] \tag{11}$$

The solitary patterns solution in a closed form of Equations (4) and (5) is given by:

$$u(x,t) = \frac{16c-1}{24}\left[\cosh\left(\frac{x}{2}\right)\cosh\left(\frac{ct^{\alpha}}{2},\alpha\right) - \sinh\left(\frac{x}{2}\right)\sinh\left(\frac{ct^{\alpha}}{2},\alpha\right) + 1\right], \quad (12)$$

where the functions $\cosh(z,\alpha)$ and $\sinh(z,\alpha)$ are defined as:

$$\cosh(z,\alpha) = \sum_{n=0}^{\infty}\frac{z^{2n}}{\Gamma(2n\alpha+1)}, \quad \sinh(z,\alpha) = \sum_{n=0}^{\infty}\frac{z^{2n+1}}{\Gamma((2n+1)\alpha+1)}$$

If we select the initial approximation $u_0(x,0) = -((16c-1)/12)\sinh^2(x/4)$, using the HPM, we get the solitary patterns soliton,

$$u(x,t) = -\frac{16c-1}{24}\left[\cosh\left(\frac{x}{2}\right)\cosh\left(\frac{ct^{\alpha}}{2},\alpha\right) - \sinh\left(\frac{x}{2}\right)\sinh\left(\frac{ct^{\alpha}}{2},\alpha\right) - 1\right] \quad (13)$$

It is interesting to point out that for the case of $\alpha = 1$, we have $\cosh(z,\alpha) = \cosh(z)$ and $\sinh(z,\alpha) = \sinh(z)$. Therefore, the solitary patterns solutions reduce to:

$$u(x,t) = \frac{16c-1}{12}\cosh^2\left(\frac{ct-x}{4}\right) \quad (14)$$

and,

$$u(x,t) = -\frac{16c-1}{12}\sinh^2\left(\frac{ct-x}{4}\right) \quad (15)$$

which coincide with the Adomian decomposition method solution obtained by Zhu *et al.* (2007).

Example 3.2
The fractional K(3, 3, 1) equation
We first consider the fractional $K(3,3,1)$ equation with initial condition:

$$D_t^{\alpha}u + (u^3)_x - (u^3)_{xxx} + u_{5x} = 0, \quad t > 0 \quad (16)$$

$$u(x,0) = \sqrt{\frac{81c-1}{54}}\cosh\left(\frac{x}{3}\right), \quad (17)$$

where $0 < \alpha \leq 1$ and c is an arbitrary constant. We construct the homotopy which satisfies the relation,

$$D_t^{\alpha}u - p\{-(u^3)_x + (u^3)_{xxx} - u_{5x}\} = 0, \quad p \in [0,1] \quad (18)$$

$$u_0(x,0) = \sqrt{\frac{81c-1}{54}}\cosh\left(\frac{x}{3}\right). \quad (19)$$

Substituting (8) into (16) and equating the coefficients of like powers of p, we get following set of differential equations:

$$\begin{aligned}
&p^0: \quad D_t^\alpha u_0 = 0, \quad u(x,0) = \sqrt{\frac{81c-1}{54}}\cosh\left(\frac{x}{3}\right),\\
&p^1: \quad D_t^\alpha u_1 = -(u_0^3)_x + (u_0^3)_{xxx} - (u_0)_{5x}, \quad u_1(x,0) = 0,\\
&p^2: \quad D_t^\alpha u_2 = -(3u_0^2 u_1)_x + (3u_0^2 u_1)_{xxx} - (u_1)_{5x}, \quad u_2(x,0) = 0,\\
&p^3: \quad D_t^\alpha u_3 = -(3u_2 u_0^2 + 3u_0 u_1^2)_x + (3u_2 u_0^2 + 3u_0 u_1^2)_{xxx} - (u_2)_{5x}, \quad u_3(x,0) = 0,\\
&\cdots
\end{aligned} \tag{20}$$

Solving the above equations, we obtain:

$$\begin{aligned}
u_0 &= \sqrt{\frac{81c-1}{54}}\cosh\left(\frac{x}{3}\right),\\
u_1 &= -\sqrt{\frac{81c-1}{54}}\frac{c}{3}\sinh\left(\frac{x}{3}\right)\frac{t^\alpha}{\Gamma(\alpha+1)},\\
u_2 &= \sqrt{\frac{81c-1}{54}}\frac{c^2}{3^2}\cosh\left(\frac{x}{3}\right)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},\\
u_3 &= -\sqrt{\frac{81c-1}{54}}\frac{c^3}{3^3}\sinh\left(\frac{x}{3}\right)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)},\\
&\cdots
\end{aligned} \tag{21}$$

and so on, in the same manner the rest of components can be obtained using Maple. Consequently, we have the first five terms (Figure 2) for solution of Equations (16) and (17) in a series form,

$$\begin{aligned}
u(x,t) = &\sqrt{\frac{81c-1}{54}}\left[\cosh\left(\frac{x}{3}\right)\left(1 + \frac{c^2}{3^2}\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{c^4}{3^4}\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + \cdots\right)\right]\\
&- \sqrt{\frac{81c-1}{54}}\left[\sinh\left(\frac{x}{3}\right)\left(\frac{c}{3}\frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{c^3}{3^3}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots\right)\right]
\end{aligned}. \tag{22}$$

Solitary patterns solution in a closed form of Equations (16) and (17) is given by:

$$u(x,t) = \sqrt{\frac{81c-1}{54}}\left[\cosh\left(\frac{x}{3}\right)\cosh\left(\frac{ct^\alpha}{3},\alpha\right) - \sinh\left(\frac{x}{3}\right)\sinh\left(\frac{ct^\alpha}{3},\alpha\right)\right], \tag{23}$$

where the functions $\cosh(z,\alpha)$ and $\sinh(z,\alpha)$ are defined as:

$$\cosh(z,\alpha) = \sum_{n=0}^{\infty}\frac{z^{2n}}{\Gamma(2n\alpha+1)}, \quad \sinh(z,\alpha) = \sum_{n=0}^{\infty}\frac{z^{2n+1}}{\Gamma((2n+1)\alpha+1)}.$$

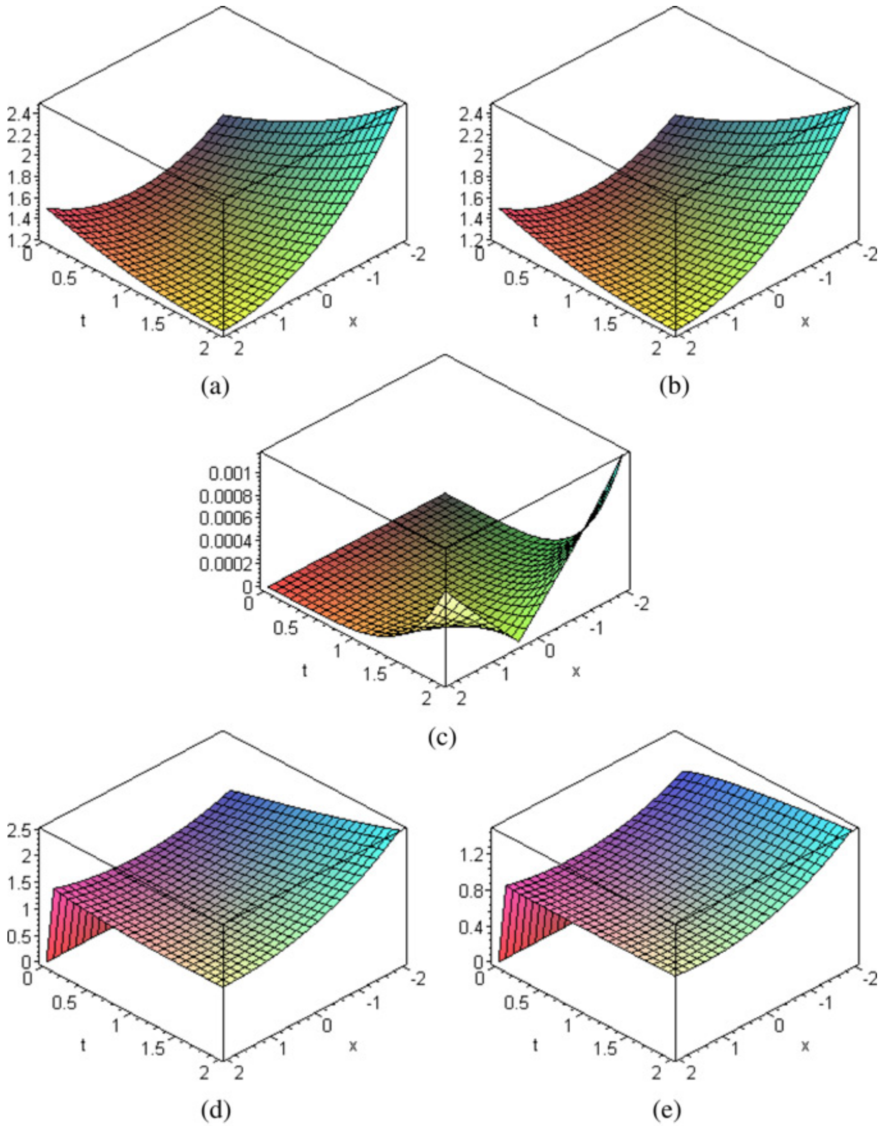

Figure 2.

Notes: $c = 1$; (a) exact solution (23) for $\alpha = 1$, (b) approximate solution (22) for, $\alpha = 1$ (c) absolute error for $\alpha = 1$, (d) exact solution (23) for $\alpha = 0.75$ and (e) exact solution (23) for $\alpha = 0.25$

If we select the initial approximation $u_0(x,0) = -\sqrt{(81c-1)/54}\cosh(x/3)$, using the HPM, we get the solitary patterns soliton

$$u(x,t) = -\sqrt{\frac{81c-1}{54}}\left[\cosh\left(\frac{x}{3}\right)\cosh\left(\frac{ct^{\alpha}}{3},\alpha\right) - \sinh\left(\frac{x}{3}\right)\sinh\left(\frac{ct^{\alpha}}{3},\alpha\right)\right] \qquad (24)$$

It is interesting to point out that for the case of $\alpha = 1$, we have $\cosh(z, \alpha) = \cosh(z)$ and $\sinh(z, \alpha) = \sinh(z)$. Therefore, the solitary patterns solutions reduce to:

$$u(x,t) = \sqrt{\frac{81c-1}{54}}\cosh\left(\frac{ct-x}{3}\right) \tag{25}$$

and

$$u(x,t) = -\sqrt{\frac{81c-1}{54}}\cosh\left(\frac{ct-x}{3}\right) \tag{26}$$

which coincide with the Adomian decomposition method solution obtained by Zhu *et al.* (2007).

4. Conclusion

In this paper, by using HPM, we successfully constructed solitary solutions for nonlinear dispersive $K(m, n, 1)$ equations with fractional time derivatives. When the fractional time derivative of order, α, $(0 < \alpha \leq 1)$ is taken as special values, our solution reduces to some known solutions in the literature. The paper shows that HPM can easily be utilized to construct solitary solutions for a broad class of nonlinear problems with fractional time derivatives.

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Corresponding author
Hüseyin Koçak can be contacted at: hkocak.ege@gmail.com